Scaling and spatial correlations in the quasibrittle process zone

Jaron Kent-Dobias    James P Sethna

Cornell University
Quasibrittle materials & fracture

Brittle with quenched disorder

Process zone of correlated microfracture, large as meters

Size and boundary effects dominate statistics of fracture

Depending on substance and scale, fracture can look clean or crumbly
Previous work suggests a *scaling crossover* between fracture regimes

Crumbly regime controlled by percolation fixed point, clean by nucleation

Avalanches dominate intermediate disorder, vanish when first fractures system

Analogous idea for crack structure: crumbly crack surface coarse-grains to clean one through microcracked crossover
Our simulations: Fuse networks

Resistive fuses as scalar elastic analogue

Quenched disorder via breaking thresholds $0 < x \leq 1$ distributed by $p(x) = \beta x^{\beta - 1}$

Fractured adiabatically: fuse with largest threshold to current ratio broken

Small $\beta$ is disordered, large $\beta$ ordered
Our simulations: Fuse networks

Resistive fuses as scalar elastic analogue

Quenched disorder via breaking thresholds
$0 < x \leq 1$ distributed by $p(x) = \beta x^{\beta-1}$

Fractured adiabatically: fuse with largest threshold to current ratio ratio broken

Small $\beta$ is disordered, large $\beta$ ordered
Our simulations: Fuse networks

Resistive fuses as scalar elastic analogue

Quenched disorder via breaking thresholds $0 < x \leq 1$ distributed by $p(x) = \beta x^{\beta - 1}$

Fractured adiabatically: fuse with largest threshold to current ratio broken

Small $\beta$ is disordered, large $\beta$ ordered
Our simulations: Fuse networks

Resistive fuses as scalar elastic analogue

Quenched disorder via breaking thresholds
$0 < x \leq 1$ distributed by $p(x) = \beta x^{\beta-1}$

Fractured adiabatically: fuse with largest threshold to current ratio broken

Small $\beta$ is disordered, large $\beta$ ordered
Our simulations: Fuse networks

Resistive fuses as scalar elastic analogue

Quenched disorder via breaking thresholds $0 < x \leq 1$ distributed by $p(x) = \beta x^{\beta - 1}$

Fractured adiabatically: fuse with largest threshold to current ratio broken

Small $\beta$ is disordered, large $\beta$ ordered
Properties of interest

Many spatial properties to study:
- backbone
- spanning cluster
- all clusters
- non-spanning clusters
- final avalanche

Focus on $g(\Delta x, \Delta y)$: probability that site displaced by $(\Delta x, \Delta y)$ is in same cluster
Properties of interest

Many spatial properties to study:

- backbone
- spanning cluster
- all clusters
- non-spanning clusters
- final avalanche

Focus on $g(\Delta x, \Delta y)$: probability that site displaced by $(\Delta x, \Delta y)$ is in same cluster
Properties of interest

Many spatial properties to study:
- backbone
- spanning cluster
- all clusters
- non-spanning clusters
- final avalanche

Focus on $g(\Delta x, \Delta y)$: probability that site displaced by $(\Delta x, \Delta y)$ is in same cluster
Properties of interest

Many spatial properties to study:
- backbone
- spanning cluster
- all clusters
- non-spanning clusters
- final avalanche

Focus on $g(\Delta x, \Delta y)$: probability that site displaced by $(\Delta x, \Delta y)$ is in same cluster
Properties of interest

Many spatial properties to study:

- backbone
- spanning cluster
- all clusters
- non-spanning clusters
- final avalanche

Focus on $g(\Delta x, \Delta y)$: probability that site displaced by $(\Delta x, \Delta y)$ is in same cluster
Properties of interest

Many spatial properties to study:

▶ backbone
▶ spanning cluster
▶ all clusters
▶ non-spanning clusters
▶ final avalanche

Focus on $g(\Delta x, \Delta y)$: probability that site displaced by $(\Delta x, \Delta y)$ is in same cluster
Properties of interest

Many spatial properties to study:

- backbone
- spanning cluster
- all clusters
- non-spanning clusters
- final avalanche

Focus on $g(\Delta x, \Delta y)$: probability that site displaced by $(\Delta x, \Delta y)$ is in same cluster
Properties of interest

Many spatial properties to study:

- backbone
- spanning cluster
- all clusters
- non-spanning clusters
- final avalanche

Focus on \( g(\Delta x, \Delta y) \): probability that site displaced by \((\Delta x, \Delta y)\) is in same cluster
Fixed points

$\beta = 0$ — isotropic, self-similar percolation

As $\beta \to 0$, $L \to \infty$, reduces (almost) exactly to percolation

- backbone — $\ell(L) \sim L^{d_{\text{min}}}$
- spanning cluster — $M(L) \sim L^{d_f}$
- clusters — $g(r) \sim |r|^{-2(d-d_f)}$
- non-spanning clusters — $n_s^c \sim s^{-\tau}$
- final avalanche — $n_s^a = \delta_{1s}$
Fixed points

$\beta = 0$ — isotropic, self-similar percolation

As $\beta \to 0$, $L \to \infty$, reduces (almost) exactly to percolation

- backbone — $\ell(L) \sim L^{d_{\text{min}}}$
- spanning cluster — $M(L) \sim L^{d_f}$
- clusters — $g(r) \sim |r|^{-2(d-d_f)}$
- non-spanning clusters — $n^c_s \sim s^{-\tau}$
- final avalanche — $n^a_s = \delta_{1s}$
Fixed points

$\beta = 0$ – isotropic, self-similar percolation

As $\beta \to 0$, $L \to \infty$, reduces (almost) exactly to percolation

- backbone — $\ell(L) \sim L^{d_{\text{min}}}$
- spanning cluster — $M(L) \sim L^{d_f}$
- clusters — $g(r) \sim |r|^{-2(d-d_f)}$
- non-spanning clusters — $n^c_s \sim s^{-\tau}$
- final avalanche — $n^a_s = \delta_{1s}$
Fixed points

$\beta = 0$ – isotropic, self-similar percolation

As $\beta \to 0$, $L \to \infty$, reduces (almost) exactly to percolation

- backbone — $\ell(L) \sim L^{d_{\text{min}}}$
- spanning cluster — $M(L) \sim L^{d_f}$
- clusters — $g(r) \sim |r|^{-2(d-d_f)}$
- non-spanning clusters — $n_s^c \sim s^{-\tau}$
- final avalanche — $n_s^a = \delta_{1s}$

\[
\begin{array}{c|c c c c c c}
\log(x) & 0.0 & 0.5 & 1.0 & 1.5 & 2.0 & 2.5 & 3.0 & 3.5 \\
\log(\log(x)) & 11.6 & 11.8 & 12.0 & 12.2 & 12.4
\end{array}
\]
Fixed points

$\beta = 0$ – isotropic, self-similar percolation

As $\beta \to 0$, $L \to \infty$, reduces (almost) exactly to percolation

- backbone — $\ell(L) \sim L^{d_{\text{min}}}$
- spanning cluster — $M(L) \sim L^{d_f}$
- clusters — $g(r) \sim |r|^{-2(d-d_f)}$
- non-spanning clusters — $n^c_s \sim s^{-\tau}$
- final avalanche — $n^a_s = \delta_{1s}$
Fixed points

$\beta = 0$ – isotropic, self-similar percolation

As $\beta \to 0$, $L \to \infty$, reduces (almost) exactly to percolation

- backbone — $\ell(L) \sim L^{d_{\text{min}}}$
- spanning cluster — $M(L) \sim L^{d_f}$
- clusters — $g(r) \sim |r|^{-2(d-d_f)}$
- non-spanning clusters — $n^c_s \sim s^{-\tau}$
- final avalanche — $n^a_s = \delta_{1s}$
Fixed points

\( \beta = 0 \) – isotropic, self-similar percolation

As \( \beta \rightarrow 0, L \rightarrow \infty \), reduces (almost) exactly to percolation

- backbone — \( \ell(L) \sim L^{d_{\text{min}}} \)
- spanning cluster — \( M(L) \sim L^{d_f} \)
- clusters — \( g(r) \sim |r|^{-2(d-d_f)} \)
- non-spanning clusters — \( n_s^c \sim s^{-\tau} \)
- final avalanche — \( n_s^a = \delta_{1s} \)
Fixed points

$\beta = 0$ – isotropic, self-similar percolation

As $\beta \to 0$, $L \to \infty$, reduces (almost)

- exactly to percolation

- backbone — $\ell(L) \sim L^{d_{\text{min}}}$
- spanning cluster — $M(L) \sim L^{d_f}$
- clusters — $g(r) \sim |r|^{-2(d-d_f)}$
- non-spanning clusters — $n^c_s \sim s^{-\tau}$
- final avalanche — $n^a_s = \delta_{1s}$
Fixed points

$\beta = \infty$ – anisotropic, self-affine nucleation

As $L \to \infty$, $\beta \to \infty$, reduces to nucleated crack propagation

- backbone
- spanning cluster
- clusters
- non-spanning clusters
- final avalanche
Fixed points

$\beta = \infty$ – anisotropic, self-affine nucleation

As $L \to \infty$, $\beta \to \infty$, reduces to nucleated crack propagation

- backbone
- spanning cluster
- clusters
- non-spanning clusters
- final avalanche

$log(g(x))$ vs $log(x)$
**Fixed points**

\[ \beta = \infty \quad \text{-- anisotropic, self-affine nucleation} \]

As \( L \to \infty, \beta \to \infty \), reduces to nucleated crack propagation

- backbone
- spanning cluster
- clusters
- non-spanning clusters
- final avalanche
Fixed points

\[ \beta = \infty \text{ – anisotropic, self-affine nucleation} \]

As \( L \to \infty, \beta \to \infty \), reduces to nucleated crack propagation

- backbone
- spanning cluster
- clusters
- non-spanning clusters
- final avalanche

\[
\log(g(x))
\]

\[
\log(x)
\]
Fixed points

\( \beta = \infty \) – anisotropic, self-affine nucleation

As \( L \to \infty, \beta \to \infty \), reduces to nucleated crack propagation

- backbone
- spanning cluster
- clusters
- non-spanning clusters
- final avalanche
Fixed points

$\beta = \infty$ – anisotropic, self-affine nucleation

As $L \to \infty$, $\beta \to \infty$, reduces to nucleated crack propagation

▶ backbone
▶ spanning cluster
▶ clusters
▶ non-spanning clusters
▶ final avalanche

\[
\begin{align*}
\log(g(x)) & \quad \log(x) \\
6 & \quad 0.5 \\
4 & \quad 1.0 \\
2 & \quad 1.5 \\
0 & \quad 2.0 \\
-2 & \quad 2.5 \\
-4 & \quad 3.0 \\
-6 & \quad 3.5
\end{align*}
\]
**Fixed points**

\( \beta = \infty \) – anisotropic, self-affine nucleation

As \( L \to \infty, \beta \to \infty \), reduces to nucleated crack propagation

- backbone
- spanning cluster
- clusters
- non-spanning clusters
- final avalanche

\[
\log(g(x))
\]

\[
\log(x)
\]
Fixed points

$\beta = \infty$ – anisotropic, self-affine nucleation

As $L \to \infty$, $\beta \to \infty$, reduces to nucleated crack propagation

- backbone
- spanning cluster
- clusters
- non-spanning clusters
- final avalanche

$\log(g(x))$

$log(x)$

0.5 1.0 1.5 2.0 2.5 3.0 3.5

-6 -4 -2 0 2 4 6
Self-similarity to self-affinity

Voltage (strain) applied along one direction—we should expect anisotropy!

Self-affine anisotropy emerges at different scales in different properties, but well within the intermediate microfractured regime.
Self-similarity to self-affinity

Voltage (strain) applied along one direction—we should expect anisotropy!

Self-affine anisotropy emerges at different scales in different properties, but well within the intermediate microfractured regime.
Self-similarity to self-affinity

Voltage (strain) applied along one direction—we should expect anisotropy!

Self-affine anisotropy emerges at different scales in different properties, but well within the intermediate microfractured regime.
**Scaling theory**

Quantities like moments of $g$ depend on $L_x \beta^\nu_x$ and $L_y \beta^\nu_y$, no simple “collapse”

Show expected isotropic percolation scaling in disordered limit, unusual crossover in the intermediate regime.

Spatial properties of avalanches remain anisotropic in all regimes.
Scaling theory

Quantities like moments of $g$ depend on $L_x \beta^\nu x$ and $L_y \beta^\nu y$, no simple “collapse”

Show expected isotropic percolation scaling in disordered limit, unusual crossover in the intermediate regime.

Spatial properties of avalanches remain anisotropic in all regimes.
Continuing work

Need to make consistent cross-property measurement of exponents; working on expected form of scaling functions through different regimes

How does the established multifractal distribution of bond currents (stresses) affect this analysis, if at all?
Continuing work

Need to make consistent cross-property measurement of exponents; working on expected form of scaling functions through different regimes

How does the established multifractal distribution of bond currents (stresses) affect this analysis, if at all?
Continuing work

Need to make consistent cross-property measurement of exponents; working on expected form of scaling functions through different regimes

How does the established multifractal distribution of bond currents (stresses) affect this analysis, if at all?
Continuing work

Need to make consistent cross-property measurement of exponents; working on expected form of scaling functions through different regimes

How does the established multifractal distribution of bond currents (stresses) affect this analysis, if at all?
Continuing work

Need to make consistent cross-property measurement of exponents; working on expected form of scaling functions through different regimes

How does the established multifractal distribution of bond currents (stresses) affect this analysis, if at all?
Continuing work

Need to make consistent cross-property measurement of exponents; working on expected form of scaling functions through different regimes

How does the established multifractal distribution of bond currents (stresses) affect this analysis, if at all?
Continuing work

Need to make consistent cross-property measurement of exponents; working on expected form of scaling functions through different regimes

How does the established multifractal distribution of bond currents (stresses) affect this analysis, if at all?
Questions?